Worksheet -- ABSOLUTE VALUE INEQUALITIES (Section 2.8) (1)

① $\left|\frac{6-2y}{y}\right| < 4$

$\frac{6-2y}{y} < 4$ AND $\frac{6-2y}{y} > -4$

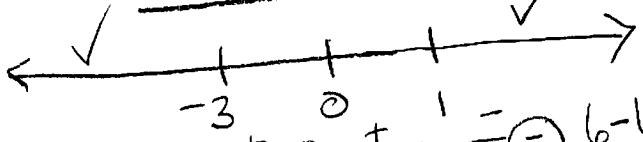
$\frac{6-2y}{y} - 4 < 0$ $\quad$ $\frac{6-2y}{y} + 4 > 0$

$\frac{6-2y}{y} - \frac{4y}{y} < 0$ $\quad$ $\frac{6-2y}{y} + \frac{4y}{y} > 0$

$\frac{6-6y}{y} < 0$ AND $\frac{6+2y}{y} > 0$

CV $6-6y=0$ $y=0$ $\quad$ $6+2y=0$
$y=1$ $\quad$ $y=-3$

$(-\infty, -3) \cup (1, \infty)$



$\frac{+}{-}\ominus$ $\quad$ $\frac{+}{-}\ominus$ $\quad$ $\frac{+}{+}+$ $\quad$ $\frac{-}{+}\ominus$ $\quad \frac{6-6y}{y} < 0$

$\frac{-}{-}\oplus$ $\quad$ $\frac{+}{-}-$ $\quad$ $\frac{+}{+}\oplus$ $\quad$ $\frac{+}{+}\oplus$ $\quad \frac{6+2y}{y} > 0$

② $\left|\frac{4+y}{y-2}\right| > 3$

$\frac{4+y}{y-2} > 3$ OR $\frac{4+y}{y-2} < -3$

$\frac{4+y}{y-2} - 3 > 0$ $\quad$ $\frac{4+y}{y-2} + 3 < 0$

$\frac{4+y}{y-2} + \frac{-3(y-2)}{y-2} > 0$ $\quad$ $\frac{4+y}{y-2} + \frac{3y-6}{y-2} < 0$

$\frac{4+y}{y-2} + \frac{-3y+6}{y-2} > 0$ $\quad$ $\frac{4y-2}{y-2} < 0$

$\frac{10-2y}{y-2} > 0$ OR

CV $10-2y=0$ $\quad$ $y-2=0$
$-2y=-10$ $\quad$ $y=2$
$y=5$

CV $4y-2=0$ $\quad$ $y-2=0$
$y=\frac{1}{2}$ $\quad$ $y=2$

$\left(\frac{1}{2}, 2\right) \cup (2, 5)$

$\frac{10-2y}{y-2} > 0$ $\quad \frac{+}{-} = - \quad \frac{+}{-} = - \quad \frac{+}{+} = \oplus \quad \frac{-}{+} = -$

OR

$\frac{4y-2}{y-2} < 0$ $\quad \frac{-}{-} = + \quad \frac{+}{-} = \ominus \quad \frac{+}{+} = + \quad \frac{+}{+} = +$

③ $\left|\frac{3-3x}{x+4}\right| \geq 5$

$\frac{3-3x}{x+4} \geq 5$ OR $\frac{3-3x}{x+4} \leq -5$

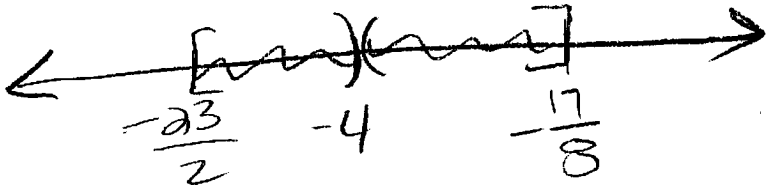
$\frac{3-3x}{x+4} - 5 \geq 0$ $\quad$ $\frac{3-3x}{x+4} + 5 \leq 0$

$\frac{3-3x}{x+4} + \frac{-5x-20}{x+4} \geq 0$ $\quad$ $\frac{3-3x}{x+4} + \frac{5x+20}{x+4} \leq 0$

$\frac{-8x-17}{x+4} \geq 0$ OR $\quad$ $\frac{2x+23}{x+4} \leq 0$

CV $x = -\frac{17}{8}$ $\quad$ CV $x = -\frac{23}{2}$ $\quad x = -4$

$\left[-\frac{23}{2}, -4\right) \cup \left(-4, -\frac{17}{8}\right]$



$\frac{+}{(-)}$ $\quad$ $\frac{+}{(-)}$ $\quad$ $\frac{+}{(+)}$ ✓ $\quad$ $\frac{-}{(+)}$ $\quad \frac{-8x-17}{x+4} \geq 0$

OR

$\frac{-}{(-)}$ $\quad$ $\frac{+}{(-)}$ ✓ $\quad$ $\frac{+}{(+)}$ $\quad$ $\frac{+}{(+)}$ $\quad \frac{2x+23}{x+4} \leq 0$

(4) $\left|\frac{2}{2-x}\right| \le 4$

$\frac{2}{2-x} \le 4$ AND $\frac{2}{2-x} \ge -4$

$\frac{2}{2-x} - 4 \le 0$ $\quad$ $\frac{2}{2-x} + 4 \ge 0$

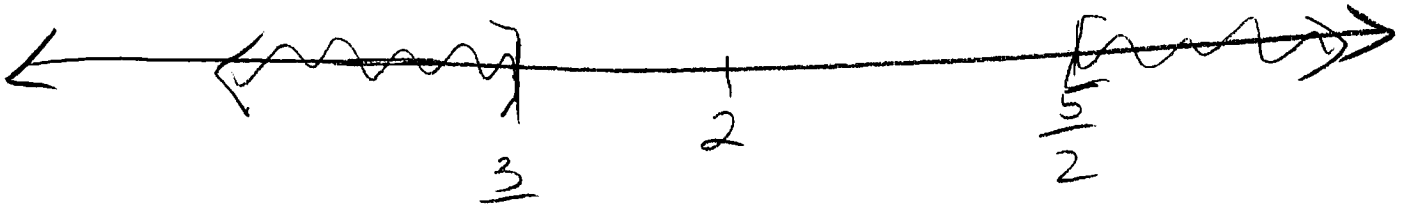
$\frac{2}{2-x} - \frac{8-4x}{2-x} \le 0$ $\quad$ $\frac{2}{2-x} + \frac{8-4x}{2-x} \ge 0$

$\frac{-6-4x}{2-x} \le 0$ $\quad$ $\frac{10-4x}{2-x} \ge 0$

$\frac{-2(3-2x)}{2-x} \le 0$ $\quad$ CV $x = \frac{5}{2}$

CV $x = \frac{3}{2}$ $\quad$ $x = 2$

$x = 2$



$\frac{-2(3-2x)}{2-x} \le 0$ $\quad$ $\frac{(-)(+)}{(+)}\ominus$ ✓ $\quad$ $\frac{(-)(-)}{(+)}\oplus$ ✗ $\quad$ $\frac{(-)(-)}{(-)}$ ✓ $\quad$ $\frac{(-)(-)}{(-)}$ ✓

AND

$\frac{10-4x}{2-x} \ge 0$ $\quad$ $\frac{(+)}{(+)}\oplus$ ✓ $\quad$ $\frac{(+)}{(+)}\oplus$ ✓ $\quad$ $\frac{(+)}{(-)}\ominus$ $\quad$ $\frac{(-)}{(-)}+$ ✓

$$\left(-\infty, \frac{3}{2}\right] \cup \left[\frac{5}{2}, \infty\right)$$

(5) $\left|\frac{5-5p}{p+2}\right| > 0$

$\frac{5-5p}{p+2} > 0$ OR $\frac{5-5p}{p+2} < 0$

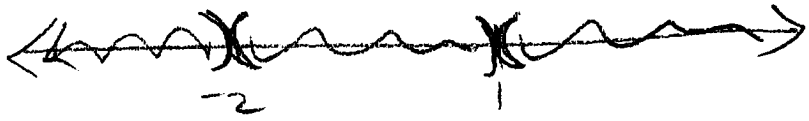
$\therefore \frac{5-5p}{p+2} \neq 0$

$5-5p \neq 0$

$-5p \neq -5$

$p \neq 1$

$p \neq -2$



$(-\infty,-2) \cup (-2,1) \cup (1,\infty)$

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(6) $|x^2-8| \geq 7$

$x^2-8 \geq 7$ OR $x^2-8 \leq -7$

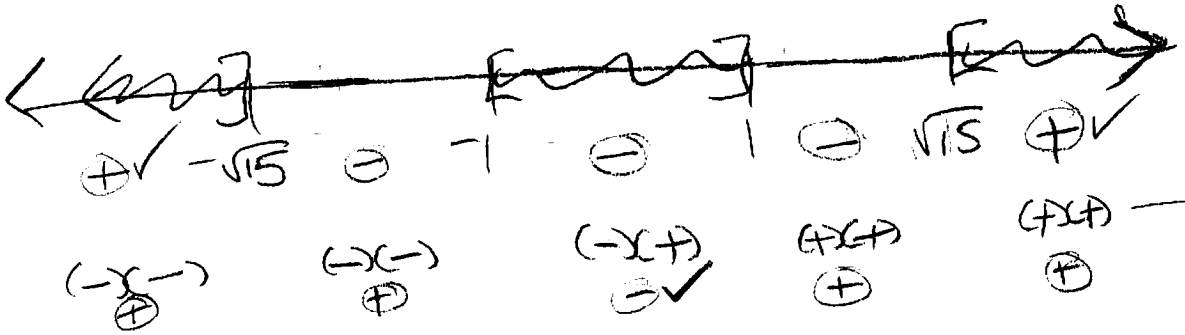
$x^2-15 \geq 0$ $x^2-1 \leq 0$

$x \geq \pm\sqrt{15}$ OR $(x-1)(x+1) \leq 0$

$x=1$

$x=-1$

$(-\infty,-\sqrt{15}] \cup [-1,1] \cup [\sqrt{15},\infty)$



Worksheet Abs. Val. Ineq. Continued

⑦ $|x^2-2x| \le 8$

$x^2-2x \le 8$ AND $x^2-2x \ge -8$

$x^2-2x-8 \le 0$

$(x-4)(x+2) \le 0$

CV $x=4$

$x=-2$

$x^2-2x+8 \ge 0$

$a=1 \quad b=-2 \quad c=8$

$x = \frac{2 \pm \sqrt{4-4(1)(8)}}{2}$

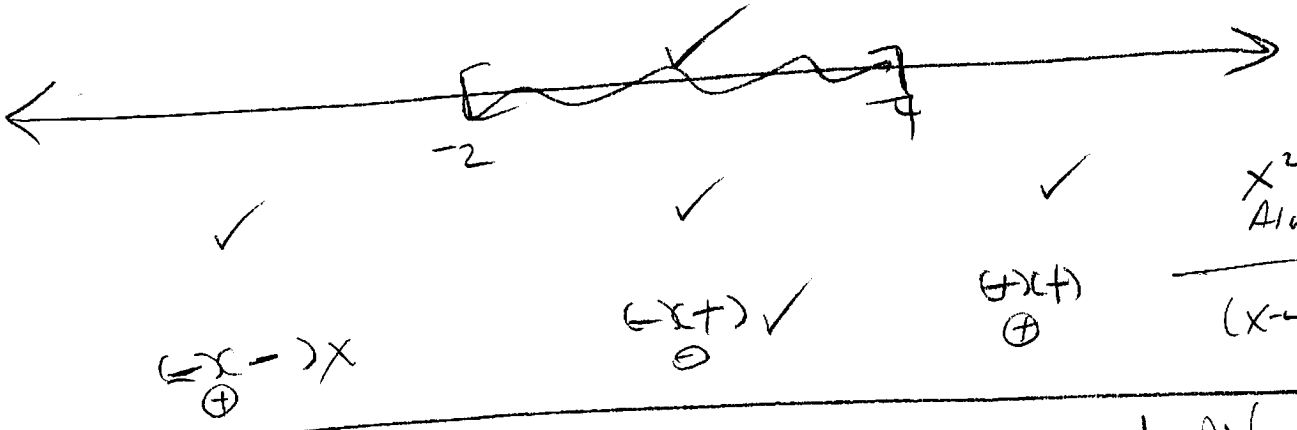
$x = \frac{2 \pm \sqrt{-28}}{2}$

No CV

Test 0

ALWAYS TRUE

$[-2, 4]$



⑧ $|3x-1| \le 2|2x+1|$ create your own rational AV ineq.

$\left|\frac{3x-1}{2x+1}\right| \le 2$

$\frac{3x-1}{2x+1} \le 2$ AND $\frac{3x-1}{2x+1} \ge -2$

$\frac{3x-1}{2x+1} - \frac{4x+2}{2x+1} \le 0$

$\frac{-x-3}{2x+1} \le 0$

CV $x=-3$

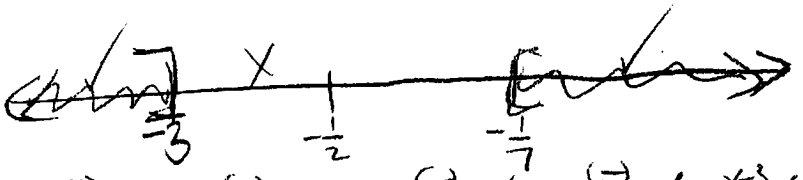
$x=-\frac{1}{2}$

$\frac{3x-1}{2x+1} + \frac{4x+2}{2x+1} \ge 0$

$\frac{7x+1}{2x+1} \ge 0$

CV $x=-\frac{1}{7}$

$x=-\frac{1}{2}$



$\frac{(+)}{(-)}\Theta$ ✓ $\frac{(-)}{(-)}\oplus$ $\frac{(-)}{(+)}\Theta$ ✓ $\frac{(-)}{(+)}\Theta$ ✓ $\frac{-x-3}{2x+1} \le 0$

$\frac{\oplus}{\ominus}$ +✓ $\frac{(+)}{(-)}$ +✓ $\frac{(-)}{(+)}$ -✗ $\frac{(+)}{(+)}$ +✓ $\frac{7x+1}{2x+1} \ge 0$

Does $-\frac{1}{2}$ belong? No

$|3(-\frac{1}{2})-1| \stackrel{?}{\le} 2|2(-\frac{1}{2})+1|$

$+\frac{5}{2} \le 2(0)$ ← Not true statement

$(-\infty, 3] \cup [-\frac{1}{7}, \infty)$

(9) $|2x-3| \geq |x+6|$

$\left|\frac{2x-3}{x+6}\right| \geq 1$

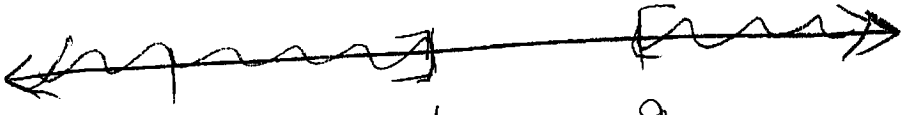
$\frac{2x-3}{x+6} \geq 1$ OR $\frac{2x-3}{x+6} \leq -1$

$\frac{2x-3}{x+6} - \frac{x+6}{x+6} \geq 0$ $\quad$ $\frac{2x-3}{x+6} + \frac{x+6}{x+6} \leq 0$

$\frac{x-9}{x+6} \geq 0$ $\quad$ $\frac{3x+3}{x+6} \leq 0$

CV $x=9$, $x=-6$ $\quad$ CV $x=-1$, $x=-6$

$(-\infty, -1] \cup [9, \infty)$



-6 $\quad$ -1 $\quad$ 9

$\frac{(-)}{(-)}$ ✓ $\quad$ $\frac{(-)}{(+)}$ $\quad$ $\frac{(-)}{(+)}$ $\quad$ ✓ $\frac{(+)}{(+)}$ $\quad$ $\frac{x-9}{x+6} \geq 0$

OR

$\frac{(-)}{(-)}$ $\quad$ $\frac{(-)}{(+)}$ ✓ $\quad$ $\frac{(+)}{(+)}$ $\quad$ $\frac{(+)}{(+)}$ $\quad$ $\frac{3x+3}{x+6} \leq 0$

check if -6 belongs because you created the rational abs value by dividing both sides by $|x+6|$

$|2x-3| \stackrel{?}{\geq} |x+6|$ $\quad$ Yes -6 belongs

$|2(-6)-3| \stackrel{?}{\geq} |-6+6|$

$|-12-3| \geq |0|$

$15 \geq 0$ True Statement

(10) $\frac{1}{|x-4|} < \frac{1}{|x+7|}$

$|x-4| > |x+7|$

$\left|\frac{x-4}{x+7}\right| > 1$

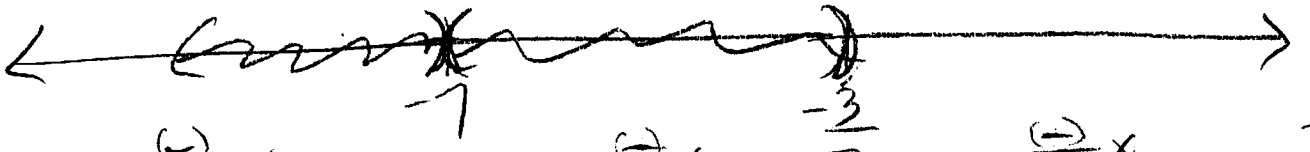
$\frac{x-4}{x+7} > 1$ OR $\frac{x-4}{x+7} < -1$

$\frac{x-4}{x+7} - \frac{x+7}{x+7} > 0$ $\quad \frac{x-4}{x+7} + \frac{x+7}{x+7} < 0$

$\frac{-11}{x+7} > 0$ OR $\frac{2x+3}{x+7} < 0$

CV = $x = -7$ $\quad$ CV = $x = -\frac{3}{2}$, $x = -7$

$(-\infty, -7) \cup (-7, -\frac{3}{2})$



$\frac{(-)}{(-)}$ ✓ $\quad \frac{(-)}{(+)}$ X $\quad \frac{(-)}{(+)}$ X $\quad \frac{-11}{x+7} > 0$

OR

$\frac{(-)}{(-)}$ X $\quad \frac{(-)}{(+)}$ ✓ $\quad \frac{(+)}{(+)}$ X $\quad \frac{2x+3}{x+7} < 0$

-7 does not belong.